Role of consciousness in two different cognitive tasks, apprehension and judgment

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Abstract

Apprehension, common to humans and animals, lasts around 1 sec. It can be explained in Bayesian terms, that is, by the formalism of classical physics. Judgment, that is, the connection between two apprehensions formalized in some language, is exclusive of humans. It violates the Leggett Garg Inequalities (LGI); this marks a quantum behaviour not describable by classical physics. LGI violation means that successive pieces of a discourse affect each other. A quantum formalism is necessary to describe the judgment. This however does not require a microphysical model, as done in some popular accounts.

Awareness of apprehensions has been taken to be consciousness. We rather call consciousness (better, self-consciousness) only the fact that the same oneself supervises both apprehensions from the comparison of which the judgment emerges. As result, ethical and esthetical decisions can not be confined within the single apprehension, as pretended in the neuro-approaches. Decisions result from a repeated comparison that stops when conformity has been reached. Such a procedure is the modern reformulation of Thomas Aquinas’ adaequatio intellectus et rei.
**Apprehension and judgment**

*Apprehension*, that is, a perception eliciting a motor response, is a task common to humans and animals. It lasts around 1 sec [1], and evidence of its occurrence can be retrieved by laboratory techniques [2]. A qualitative diagram of the brain operations is sketched in Fig.1. Apprehension can be explained in Bayesian terms (Fig.2-top), that is with classical statistics as purported in the 1700’s by Bayes and later formalized by Boole in the 1800’s and Kolmogorov in the 1900’s. Thus, it is described by the formalism developed for classical physics.

On the other hand, *Judgment*, that is, the connection between two independent perceptions formalized in some language, is exclusive of humans. It requires a time roughly thrice that of apprehension [3] (Fig.2-bottom). Furthermore, it violates the Leggett Garg Inequalities (LGI) [4]. LGI are the time equivalent of the Bell inequalities [5], the violation of which marks a quantum behaviour not describable by classical physics.

Evidence of such a LGI violation has just been reported [6]. It means that successive pieces of a discourse affect each other, that is, the corresponding apprehensions violate the requirement of non invasive measurement (NIM), which is a necessary requisite of LGI. It follows that the formal description of a discourse can not be done in terms of disjoint logical sets (Boole space of classical physics) but rather in terms of vectors in a complex Hilbert space. Hence the quantum formalism, historically introduced to describe micro-physics, is necessary to describe the formulation of a judgment. This results from the LGI violation, without any need to recur to microphysical models as done in some popular accounts of cognition.

In a previous work [7], before having reached evidence of LFI violation, a qualitative account of the judgment process was called “inverse Bayes process”.

Thus far, apprehension is accompanied by *awareness* of its contents and this has been currently taken to be *consciousness*, raising the problem of *qualia* and being the source of all debates between first- person and third-person accounts.

At variance with this current trend, we think it more appropriate to call consciousness (or better *self-consciousness*) only the realization that the same oneself supervises both apprehensions that are being compared, and from the comparison of which the judgment emerges.

As result of this approach [6], ethical and esthetical decisions can not be confined within the single apprehension, as presumed in the so-called *neuro-approaches* that try to provide evidence of a human decision in terms of NCC. They rather result from a repeated comparison that stops when an adequate conformity has been reached between the pieces under comparison. Such a conformity procedure is the modern reformulation of what had been called by Thomas Aquinas as *adaequatio intellectus et rei*.

**Creativity and quantum aspects**

This ability to compare different apprehensions, going beyond the emotions aroused by each one separately, should be considered as a mark of *creativity*. Creativity is associated with the build up of new connections between different apprehensions, going beyond the expectations of a Bayesian approach [7].

We illustrate in detail what we mean by *creativity*. The apprehension is explained in terms of Bayes interference, as illustrated in detail in Fig.2-top. A sensorial stimulus $d$ must be interpreted in order to elicit an appropriate motor response. This takes place by formulating a set of prior probabilities $h$; each $h$ has an associated expected stimulus $d$ via an interpretational model $P(d|h)$ (read: conditional probability of occurrence of $d$, given $h$). Since however $d$ is given, the most appropriate posterior probability $h^*$ emerges via the Bayes formula reported in the figure on the left.

On the contrary, in the judgment (Fig.2-bottom) we compare two linguistic lumps (say, two verses of a poem) which have been acquired as separate perceptions. The previous one (called $h^*$) is
retrieved by memory in order to be compared with the second one (called \(d\)). More precisely, the corresponding probabilities are given, and they can be introduced into the Bayes equation to extract what before was preassigned and that now results aposteriori, that is, the conditional probability of \(d\) given \(h^*\). Thus the interpretational model \(P(d|h)\), that in the apprehension is presupposed, in the judgment is built as result of the comparison.

This fact entails two important results, namely, i) the judging subject must be aware of being the same self exposed to \(h^*\) and \(d\) in order to extract a sound comparison (self-consciousness); ii) \(P(d|h)\) results from the judgment, it not pre-assigned; hence, it is the result of a non-algorithmic operation that we call creativity.

Fig 3 represents the standard Bayes inference and the inverse Bayes inference in a probability space. Figs 2 and 3 provide a phenomenal account of the role of judgment in matching the mutual relations of different lumps of a linguistic flow. A more rigorous account of this role emerges from the evidence of the transient LGI violation [6]. This means that at 2-3 sec separation between successive apprehensions and for a window of 1 sec, a quantum search holds like in a quantum computer, yielding a dramatic speed up in processing complex situations with respect to a classical computer.

We recall that a classical problem with \(N\) objects, each one with 2 degrees of freedom is embedded in a \(2N\)-dimensional space; whereas a quantum problem with the same number \(N\), if each one acts as a q-bit, lives in \(2^N >> 2N\) dimensional space. This fact opens new possibilities not accessible in the classical realm. Let us display an example: if \(N=10\), the classical event space, that contains all mutual relations among the 10 objects, has 20 dimensions; instead, the quantum space has \(2^{10}=1024\) dimensions, thus the number of the possible relations is much higher. If we call complexity the number of possible situations emerging from the interaction of these objects, this complexity is overwhelmingly higher in the quantum case[8]. This means that a Bayesian agent can formulate a rather modest set of expectations about its future as compared to a quantum agent.

We thus put forward the working hypothesis that the transient quantum aspect of a judgment is the source of the “swap of model” drawn in Fig.3-bottom, that is, of the non algorithmic jump called creativity that marks the difference between a classical computation and a human cognitive performance.

This pictorial representation explains the meaning of 1931 Goedel incompleteness theorem, that is a fundamental limitation of any formal theory (Fig. 4).

**Conclusion**

In conclusion, creativity has been characterized as going well beyond the limited number of possibilities accessible to a Bayesian agent (as a computer or a robot). The evidence of a transient quantum region in a linguistic endeavour provides an exponential increase of the possibilities of relating different apprehensions in a judgment. This fact provides a root for free will, beyond the constrained behavior of a Bayesian agent.

In the Appendix, I report excerpts from Ref.[6], including four figures that provide evidence of LGI violation in cognitive tasks for several subjects.

**References**


6- Arecchi, F.T., Farini, A. & Megna, N. Violation of the Leggett-Garg inequality in cognitive processes (submitted for publication)


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**Fig.1.** Qualitative diagram of how the brain operates in presence of a visual stimulus. The activation of the eye’s retina is transferred to some detectors (first cortical stages) specialized to respond to particular geometries (horizontal versus vertical lines) or colors. The outputs of this stage are bottom-up signals that must be “interpreted.” They reach some higher cortical stages where also other inputs are provided by the semantic memory (top down) as possible interpretations. The top down perturbation is varied under the monitoring of a Focal attention system which provides an OK as soon as the top down expectation matches the features of the bottom up stimulus. At this point the perception is completed; it is an apprehension that elicits an appropriate motor (or also linguistic in the human case) response.
APPREHENSION (Bayes inference)
Selection of $h^*$ from $h$, by joint action of
stimulus $P(d)$ (bottom-up) and model $P(d|h)$ (top-down)

$$P(h^*) = P(d|h) P(h) / P(d)$$

JUDGMENT (inverse Bayes)
Comparison of $d$ with $h^*$,
whereby the most adequate model emerges

$$P(d|h^*)$$

Fig. 2-top) APPREHENSION-How the most plausible hypothesis $h^*$ is selected out of many guesses $h$ by joint action of a sensorial stimulus $d$ spread over a probability $P(d)$ and an interpretational model stored in the memory consisting of the conditional probability $P(d|h)$ that $d$ emerges once given $h$.

Fig. 2-bottom) JUDGMENT- As one compares two different apprehensions, inserting the respective probabilities $P(h^*)$ and $P(d)$ into Bayes formula written in the top part, the conditional $P(d|h^*)$ emerges as a result. It can be read as the most appropriate $d$ once given $h^*$. 
Fig. 3-top) Successive applications of the Bayes process within a single model. The updated probabilities are like points on a probability mountain. The climbing up is an asemiotic procedure driven by fixed algorithm (the model); the procedure can be done by a computer.

Fig. 3-bottom) Going beyond a single model is equivalent to changing the mountain to be climbed. The corresponding horizontal jump (swap of model) is non-algorithmic; this is what we call creativity.
**Fig. 4-** Goedel incompleteness theorem-1931- states that in a theory, starting from a set of axioms, the truths decidable by the rules of logic (the theorems) are not all the truths, but there are more truths not deducible via the formalism. Using the picture introduced in the previous figure, we see immediately that a creative mathematician can climb mountains other than the Bayesian one, where the formalism acts as a set of fixed rules limiting the possible consequences of the axioms.

**Appendix- Violation of the Leggett-Garg inequality in cognitive processes**

(from Ref. 6)

Violation of the Bell inequality has been considered a distinctive sign of the quantum entanglement. A Bell-type experiment consists of the comparison of data collected simultaneously in different locations. Leggett and Garg have introduced an equivalent time dependent inequality (LGI) in the case of a single measurement channel monitored at different times. LGI is based on two assumptions, both necessary, namely, macrorealism (MR) and non-invasive measurement (NIM). In cognitive tasks, there are instances where NIM is not a valid assumption; in such cases one should expect LGI violation. Here we report LGI violation in cognitive tasks consisting in the identification of mutually incompatible words with negligible semantic content; the violation is maximal at an inter-stimulus time \( \tau_{LG} \) close to, but consistently lower than, the characteristic times associated with other, semantically rich, linguistic endeavours. The LGI violation persists over a time window of 1 sec around \( \tau_{LG} \); outside this window NIM holds.

At variance with the apprehension, the judgment is strongly contextual; this suggests that judgments should violate the Leggett Garg inequality (LGI).

To avoid any semantic bias, we rely on the most elementary lexicon, namely, a sequence of binary words with negligible semantic flavour. Precisely, we expose several normal subjects to a balanced presentation of a Necker cube and request to press two different buttons, depending on the cube face perceived as the front one, anytime a short audio signal alerts the subject (fig.5).
The simplest form of the LGI derives from examining a system that has only two possible states, to which we attribute respectively measurement values $Q=\pm1$. The key here is that we have measurements at two different times, and one or more times between the first and last measurement. The simplest example is where the system is measured at three successive times $t_1 < t_2 < t_3$. For instance, the correlation $C_{13}$ between times $t_1$ and $t_3$, for $N$ realizations of the experiment, reads

$$C_{13} = \frac{1}{N} \sum_{r=1}^{N} Q_r(t_1)Q_r(t_3)$$

We define $K$ as $K = C_{12} + C_{23} - C_{13}$. In terms of actual measurements $Q$’s, $K$ is given by

$$K = \frac{1}{N} \sum_{r=0}^{N} (Q(t_1)Q(t_2) + Q(t_2)Q(t_3) - Q(t_1)Q(t_3))_r$$

For every realization $r$, the term in the parentheses must be less than or equal to unity, so that the result for the sum is also less than (or equal to) unity. Thus the three time LGI is

$$K \leq 1$$

In the derivation, it has been assumed that the quantity $Q$, representing the state of the system, always has a definite value (macrorealism per se) and that its measurement at a certain time does not change this value nor its subsequent evolution (noninvasive measurability). A violation of LGI implies that at least one of these two assumptions fails.

In fig. 6 we report $K$ versus the inter-stimulus interval (ISI) $t_2-t_1=t_3-t_2$ for seven subjects. Here and in the successive figures, the horizontal straight line at $K=1$ is the upper boundary for any $K$ obtained in situations obeying LGI. For each subject, successive presentations of the sequence of fig.4 are separated by a blank time of 5 sec, well above the maximum ISI explored. In fig.4 the time duration of each cube presentation is 0.3 sec. In fact, even a continuous presentation of the cube yields similar results; indeed, the measurement times are selected by the short acoustic signals.

Reporting at each ISI value the average among the seven subjects, we have a meta-subject response, displaying a violation of LGI above a few standard deviations(Fig 7a). We make a different presentation, aligning the peaks at a time $t=0$ and plotting the averaged $K$s for negative and positive time separations from the peak $\tau_{LG}$ (fig. 7b). In Fig 7c) we plot the statistical distribution of $\tau_{LG}$ for the seven subjects.

We comment on this LGI violation around 2 sec. A very striking new fact is that the peak $\tau_{LG}$ of LGI violation is close to the maximal value of the statistical distribution of pauses in several linguistic endeavors, as e.g. the sequence of pauses in declaiming a poem (fig 8a) or performing a musical piece (fig 8b).

From the neuroscientific side, no attempt has been made to tackle the sharp discrepancy between apprehension times, for which several NCC have been devised and the judgment times necessary to build meanings out of sequences of consecutive linguistic passages.

Let us speculate about the time location $\tau_{LG}$ of the maximal LGI violation. We here formulate a tentative hypothesis: since $\tau_{LG}$ corresponds to the maximal perturbation that a word exerts on a later word (maximal deviation from NIM), then the human linguistic processing is organized so that $\tau_{LG}$ is the most appropriate coupling time between two successive groups of words. A time window of about 1 sec surrounds $\tau_{LG}$; it represents the interval over which LGI is
violated. Our cognitive abilities seem geared in such a way that in that interval NIM is suspended and two adjacent groups of words within that window interact in a non-Bayesian way. Away from that window, that is, for shorter and longer separations of word groups, a classical Bayesian strategy applies. This holds for any linguistic endeavor. Figs 8 a) and b) offer two examples of statistical distribution of pauses in the word or note flow. In both cases, the statistics have a maximum around 3 sec. We hypothesize that, in order to make sense of a text, two successive verses of a poem or measures of a symphony must be separated by a time of the order of $\tau_{LG}$.

It remains to explain why $\tau_{LG}$ in the above mentioned semantic tasks is close, but consistently larger, than that reported in our tests. We put forward the following tentative explanation. LGI violation is a strategy to correlate successive linguistic lumps one another, overcoming (or, better to say, re-adjusting) the emotional arousal that each piece provokes by itself. Our cognitive abilities seem to be geared for two different tasks, namely, i) within the single lump, make the best use of the incoming sensorial information (apprehension) and ii) combine the current apprehension with the correlation to the apprehension of a previous lump of some linguistic sequence (judgment). The two tasks may interact with different weights depending on the semantic content of the sequence we are exposed to. Task ii) corresponds to an invasive measurement, hence it implies violation of LGI. The $\tau_{LG}$ measured by us in a sequence with no semantic content provides the first preliminary evidence of mechanism ii).

LGI violation is not accounted for by the Boolean set theory used to describe classical physics. It is instead accounted for by a quantum formalism. Our report provides the laboratory evidence of a time window within which classical physics fails and a quantum description is compulsory to explain cognition. This has nothing to do with microscopic physical effects, as naively claimed in some popular accounts of cognitive phenomena.

**Fig. 5-LGI measurement procedure**

a) The Necker cube

b) Experimental procedure. Sequence of three successive presentations of the Necker cube (represented as square pulses, lasting 0.3 sec each) separated by time intervals $t_2-t_1=t_3-t_2$, adjustable from 1 sec up. The vertical arrows denote a sharp acoustic signal that demands the subject to press either button corresponding to the perceived front face of the cube. The circles denote the presentation of the cube in the absence of the acoustic signal. The three sequences correspond to $C_{12}$, $C_{23}$ and $C_{13}$ respectively. The sequences are repeated after a time much longer than $t_2-t_1$. 

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Fig. 6- K values for one subject
K values corresponding to the six selected interstimulus intervals for one of the seven subjects that participated to the experiment (see individual results in Supplementary Information). Gray horizontal lines indicate the LG limit K=1.

In order to calculate K, we attribute the value Q=+1 to the upward-right front face and Q=−1 to the downward-left front face. The correlation C12 results as a sum of products Q1*Q2, similarly for C23 and C13. By these data introduced into Eqs. (1) and (2), we evaluate K for each ISI. Error bars have been calculated as $\sqrt{3/N}$, where N = 45 is the number of trials for each interstimulus interval.

Fig. 7- K values for a meta-subject
a) K values in correspondence of the six tested interstimulus intervals. These values have been obtained by pooling data from each observer into a unique meta-observer. LG violation is evident at ISIs around 2 and 2.5 seconds, with K values above the LG limit by a few standard deviations. Error bars are calculated as in Figure 6, but with N = 45x7 (n° of trialsxn° of subjects) = 315.
b) Data for the seven subjects pooled around the peak time, here denoted as t=0. The LGI violation lasts for a transient window of duration 1 sec. Error bars are calculated as in Figure 6a, but the extreme points at t=-1.5 and t=1.5 correspond to N<315.
c) Distributions $\tau_{LG}$ of the maxima of K. $\tau_{LG}$ is peaked at 2 seconds and lasts about 1 second.
Fig. 8-Statistics of pauses in linguistic endeavours
a) Time distribution of pauses in a poetical text (Dante’s Comedy, Canto XXXIII of Inferno, played by the speaker Roberto Benigni); b) Time distribution of pauses in a musical text (intervals in the first movement of Beethoven’s Fifth Symphony, interpreted by the Symphonieorchester des Bayerischen Rundfunks directed by Leonard Bernstein).

The gray lines are Gaussian fits of the data points of a single subject. It results: a) mean=3.63 sec; standard deviation=1.15 sec; b) mean=3.18 sec; standard deviation=0.5 sec. No appreciable differences of means and standard deviations occur for different listeners.